

Threshold Effects and Perturbative Unification

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We discuss the effect of the renormalization procedure in the computation of the unification point for running coupling constants. We explore the effects of threshold-crossing on the β -functions. We compute the running of the coupling constants of the Standard Model, between m_Z and M_P , using a mass dependent subtraction procedure, and then compare the results with \overline{MS} , and with the θ -function approximation. We also do this for the Minimal Supersymmetric extension of the Standard Model. In the latter, the bounds on susy masses that one obtains by requiring perturbative unification are dependent, to some extent, on the procedure.

The last years have seen a revival of the study of scenarios for a possible perturbative unification of the strong, electromagnetic and weak interactions into a simple group G , the simplest candidate being $SU(5)$ [1,2]. In this way, the three couplings of the Standard Model (SM), g_3, g_2, g' , for $SU(3)_c \times SU(2)_L \times U(1)_Y$, would evolve up to a common value g at the unification scale M_X (order of 10^{16} GeV) from their disparate value at m_Z . Thus the study of the evolution of these couplings from low to high energies, can be used to obtain information about the possible validity of the perturbative unification picture.

Since we pretend to compare experimental values with theoretical predictions for the coupling constants g_i , how to actually study the scale evolution of the couplings becomes an issue. The tool to carry this out is the renormalization group (RG), with β_i -functions defined as $\beta_i = \frac{dg_i}{d\ln\mu}$, from which we can get $g_i(\mu)$, μ being the energy scale at which we probe the system, and g_i the renormalized coupling. The physical coupling constants are measured at the laboratory at a given energy scale μ_0 , typically of order of m_Z . We *identify* these values with $g_i(\mu_0)$, and with the help of the RGE we infer the evolution of the couplings as the scale changes. However, in order to compute the β_i -functions, one needs to specify a renormalization *scheme*, and beta functions differ for one scheme to another. Broadly speaking, one has generally two choices of scheme at his/her disposal: the modified minimal subtraction scheme (\overline{MS}) [3], and mass dependent subtraction procedures (MDSP) [4]. In \overline{MS} schemes, the β_i -functions depend on the particle content at the energy scale on which one computes them and not explicitly on the masses nor energy scale. On the other hand, MDSP schemes take into account a dependence on particle masses: each graph including particles with mass m_i running in the loop, contains a function of the ratio μ^2/m_i^2 associated with it, where μ is the energy scale. The decoupling theorem [5] ensures that the contributions from graphs with heavy field loops are suppressed at low momenta (much smaller than the masses involved). Naturally, the functions $f(\mu^2/m_i^2)$ associated with each graph have the property that $f(\mu^2/m_i^2) \rightarrow 0$, when $\mu^2/m_i^2 \rightarrow 0$, and therefore the decoupling of the heavy degrees of freedom is explicitly enforced. Because of their origin, these functions include the threshold effects arising as μ becomes larger than the putative

particle mass.

In addition to the above schemes, one can approximate the threshold effects by Heaviside- θ functions and impose this on the \overline{MS} β -functions, which are then integrated to give the effective couplings. Hence a massive particle only contributes at scales larger than its mass [6]. However, this is only an *approximation* to the contribution given by the functions $f(\mu^2/m_i^2)$ in the MDSP, and the results for the effective gauge couplings can be appreciably different (the more thresholds are crossed, the more different are the effective couplings).

To *illustrate* the relevance of the scheme-choice we calculate $\alpha_e(m_W)$, integrating the β -function from m_e with $\alpha_e^{-1}(m_e) = 137.036$, to $m_W = 81 \text{ GeV}$ and assuming $m_t \geq m_Z$, using the three methods, *viz.*, (a) \overline{MS} without any reference to the thresholds (which of course gives extremely inaccurate results as it is shown below), (b) θ -functions, and (c) MDSP. It is straightforward to see that:

$$\alpha_e^{-1} |_{\overline{MS}} = 120.06, \quad \alpha_e^{-1} |_{\theta} = 128.54, \quad \alpha_e^{-1} |_{MDSP} = 129.33.$$

Because thresholds modify the derivatives of the couplings (β -functions) and we are integrating over many decades in momenta, the effect due to the decoupling of massive degrees of freedom *and* the subtraction procedure could be important, and in any case one cannot disregard the effects of the thresholds.

A further uncertainty is introduced when making considerations relative to unification, since the choice of the unifying group, G , impacts on the renormalization of the couplings. Let us assume that this group breaks into the SM at scale M_X . Since the particle content of G is different from the SM we will have, in general, light fields (particles of the SM) and heavy fields (with masses of the order of the unification scale M_X). At low energies these heavy fields decouple from the theory. To take into account this decoupling in a proper way one can, *e.g.*, integrate out the heavy fields from the action [7,8]. Carrying this out, the simple unification condition $g_i(M_X) = g(M_X)$, is modified into $g_i(\mu)^{-2} = g(\mu)^{-2} - \lambda_i(\mu)$, where the functions λ_i depend on the masses of the heavy fields, and the interpolating scale μ satisfies $m_i \ll \mu \ll M_X$. Again the *values* of the masses are important.

We are interested in studying how the presence of different masses affects the running of the couplings (including threshold effects), as well as its consequences on a possible unification scenario for the SM. Therefore we will study here the SM with a mass dependent renormalization procedure (MDSP). The relevant masses for the problem are m_Z, m_W, m_t and m_h , the Higgs mass. We do not consider the remaining fermion masses because they are much smaller than m_Z , so that for scales $\mu \geq m_Z$ we can regard all the fermions as being massless except for the top quark.

In the symmetric phase, when $\mu \gg m_Z$, the couplings are g_3^2, g_2^2, g'^2 of $SU_c(3) \times SU_L(2) \times U_Y(1)$; when the symmetry breaks down, the relevant couplings are instead g_3^2, g_2^2 and e^2 , with the coupling g'^2 is given as a function of g_2^2 and e^2 by

$$\frac{1}{g'^2} = \frac{1}{e^2} - \frac{1}{g_2^2} . \quad (1)$$

In the laboratory we can measure g_3^2, g_2^2 and e^2 but not g'^2 . Therefore we will *calculate* the evolution of these three couplings and *infer* the evolution of g'^2 from (1).

We need to compute the β_i -functions for g_3^2, g_2^2 and e^2 in an MDSP. In general, a renormalized coupling constant, g_r^2 , is related to the bare constant g_0^2 by $g_r^2 = Z g_0^2$, where Z is a product of renormalization constants. A choice of subtraction procedure as well as an arbitrary subtraction point μ , are implicit in the computation of the Z 's. In an MDSP there are two contributions to Z : the pole part, typical of the \overline{MS} procedure, *and* finite contributions which depend on both the masses of the virtual degrees of freedom and the subtraction point. Since the β -function is defined as

$$\beta = g_r^2 \frac{d \ln g_r^2}{d \ln \mu} = g_r^2 \frac{d \ln Z}{d \ln \mu} , \quad (2)$$

this dependence on the masses and the subtraction point carries over to the β -functions. Now, for a general gauge theory which includes fermions and scalars, extra complications arise: there will be different kinds of vertices involving the coupling constants (fermion-boson and scalar-boson vertex in the case of abelian gauge theories, and also three and four boson vertices for non-abelian gauge theories). For each of these vertices, Z is given as a product of different renormalization constants Z_i :

$$Z = Z_{ex.leg} Z_v^{-1} Z_3^{\frac{1}{2}} . \quad (3)$$

Here Z_3 is the wave function renormalization constant (WFRC) of the relevant gauge boson; Z_v is the vertex renormalization constant; $Z_{ex.leg}$ are the WFRC of the external legs. The Ward–Takahashi identities guarantee the existence of relations among the infinite parts of different Z_i , in such a way that the infinite part of Z does not depend on the vertex we choose. But, as it is well known, due to the presence of masses, this is not true for the finite part: in each vertex the masses involved are different. Yet another problem arises due to the gauge dependence of the Z_i ; although each Z_i is gauge dependent, their product, Z , cannot depend on the choice of gauge since g_r^2 must be gauge independent. Again, this is true for the gauge dependence of the infinite part, but it is not true in general for the finite contributions. Therefore, in order to get an acceptable g_r^2 in an MDSP we have to define Z in a *universal* (independent of the kind of vertex), and gauge-independent way. This is easiest for abelian couplings, since in this case one can extend the abelian Ward identity, $Z_{ex.leg} = Z_v$, to the finite parts, in such a way that $g_r^2 = Z_3 g_0^2$, Z_3 being both gauge independent and *universal*. This definition of g_r^2 is equivalent to defining an effective coupling via [9]:

$$\frac{1}{g_{eff}^2} = \frac{1}{g_0^2} + \Pi_B , \quad (4)$$

where Π_B is the transverse *bare* vacuum polarization.

But this no longer works in the broken phase of non-abelian theories. Because of tree level mixing of the neutral gauge bosons of $SU(2)$ and $U(1)$, a calculation of radiative corrections to the proper self energies, $\Pi_{(Z^0)}^{\mu\nu}$ and $\Pi_{(A)}^{\mu\nu}$, at 1-loop order, gives a contribution $\Pi_{(ZA)}^{\mu\nu}$ which is (i) not purely transverse and (ii) it further mixes the self-energies. In an self-explanatory notation:

$$\Pi^{\mu\nu}(q^2) = (g^{\mu\nu} q^2 - q^\mu q^\nu) \Pi^T(q^2) + g^{\mu\nu} \Pi^L(q^2) , \quad (5)$$

where $\Pi^L(q^2)$ is in general proportional to a mass squared. The longitudinal term only appears when the symmetry is broken, since $\Pi^{\mu\nu}$ is purely transverse in an unbroken gauge

theory. Then, if the longitudinal part Π_{ZA}^L contributes to the mass matrix of the (Z^0, A) system, the photon would not be massless at 1-loop order! To maintain a massless photon without further field redefinitions one needs to eliminate the contribution from Π_{ZA}^L .

These problems affect the couplings e^2 and g_2^2 , since they reflect the non-abelian nature of the parent gauge theory. Kennedy and Lynn [10] have shown how to solve them, and, at the same time, define a $g_{r_i}^2$ appropriate to phenomenology. They split the vertex contribution for g_2^2 into a finite process-dependent part, and a “universal” part Γ' , consisting of an infinite term (dictated by the Ward identities) and a finite term to be determined¹. As pointed out by these authors, all the gauge dependence is included in the universal part. Before breaking the $SU(2)_L \times U(1)_Y$ symmetry, the g_2^2 coupling is redefined into

$$\tilde{g}_2^2 = g_2^2(1 + g_2^2\Gamma') \quad . \quad (6)$$

When the symmetry is broken we can write the electric charge in terms of \tilde{g}_2^2 and g'^2 as

$$\frac{1}{e^2} = \frac{1}{\tilde{g}_2^2} + \frac{1}{g'^2} \quad . \quad (7)$$

The advantage of working with the \tilde{g}_i^2 is now clear, since all non-abelian effects are now dumped into Γ' , and one may define effective charges associated with the bare tilde parameters in exactly the same way as it was done in the abelian case², *i.e.*

$$\frac{1}{g_i^2(q^2)} = \frac{1}{\tilde{g}_i^2} + \Pi_B^i(q^2) \quad . \quad (8)$$

For the couplings g_2^2 , e^2 we get the effective couplings:

$$\frac{1}{g_i^2(q^2)} = \frac{1}{g_i^2} + (\Pi_B^i(q^2) - 2\Gamma'(q^2)) \quad . \quad (9)$$

¹ From now on, the term “vertex” will refer to the contributions due to both the vertex itself and external legs.

² Notice that the Π_B^i refers only to the transverse part

At the same time, if we calculate the 1-loop corrections to the mass matrix of the neutral bosons, the non diagonal term will be equal to $\Pi_{ZA}^L + m_W^2 \Gamma'$, and this term is now *finite*. The finite part of Γ' may be chosen so that:

$$\Pi_{ZA}^L + m_W^2 \Gamma' = 0 \quad . \quad (10)$$

Thus (a) the photon remains massless (at least to 1-loop) and (b) we have available scheme-independent effective charges³.

For the $SU(3)$ coupling the equivalent of (9) reads:

$$\frac{1}{g_3^2(q^2)} = \frac{1}{g_3^2} + (\Pi_B^{(3)}(q^2) - 2\Gamma^{(3)}(q^2)) \quad , \quad (11)$$

where $\Pi_B^{(3)}(q^2)$ is the bare proper self-energy of the gluon, and $\Gamma^{(3)}(q^2)$ is the universal part of the gluon vertex. Since gluons are massless, the process-independent part of the *vertex* consists of an infinite and gauge dependent term, while $\Pi_B^{(3)}$ includes the contributions due to massive quarks.

The above equations for the effective charges can be written in terms of the couplings at scale m_Z in the form:

$$\frac{1}{g_i^2(q^2)} = \frac{1}{g_i^2(m_Z^2)} + (\Pi_B^i(q^2) - \Pi_B^i(m_Z^2) - 2\Gamma'(q^2) + 2\Gamma'(m_Z^2)) \quad ; \quad (12)$$

the combinations $\Pi_B^i(q^2) - \Pi_B^i(m_Z^2)$, $\Gamma'(q^2) - \Gamma'(m_Z^2)$ that appear in (12) are now finite. Using dimensional regularization we calculate Π_B^i, Γ' ; in the Landau gauge we get the following expressions:

$$(4\pi)^2 \Gamma' = \frac{3}{2} \left(\frac{2}{\varepsilon} - \ln \frac{m_W^2}{\mu^2} \right) + F_\Gamma(a_W) \quad , \quad (13)$$

³ It may be shown that (9) remains gauge independent when coupled to the RGE for the gauge parameter α ; now, since $\alpha = 0$ is a fixed point of this RGE, we will choose to work in the Landau gauge where the value of α stays at zero, and therefore Goldstone bosons and ghosts remain massless.

$$\begin{aligned}
\Pi^A &= \Pi_{WW}^A + \Pi_{W+}^A + \Pi_{++}^A + \Pi_f^A \quad , \\
(4\pi)^2 \Pi_{WW}^A &= - \left\{ \frac{13}{3} \left(\frac{2}{\varepsilon} - \ln \frac{m_W^2}{\mu^2} \right) + F_1(a_W, a_W) \right\} \quad , \\
(4\pi)^2 \Pi_{W+}^A &= -2a_W F_2(a_W, 0) \quad , \\
(4\pi)^2 \Pi_{++}^A &= \frac{1}{3} \left(\frac{2}{\varepsilon} - \ln \frac{-p^2}{\mu^2} + \frac{8}{3} \right) \quad , \\
(4\pi)^2 \Pi_f^A &= \frac{4}{3} \sum_f Q_f^2 \left(\frac{2}{\varepsilon} - \ln \frac{m_f^2}{\mu^2} + F_f(a_f, a_f) \right) \quad ,
\end{aligned} \tag{14}$$

$$\begin{aligned}
\Pi^W &= s_\theta^2 \Pi_{WA}^W + c_\theta^2 \Pi_{WZ}^W + \Pi_{Wh}^W + s_\theta^2 \Pi_{A+}^W + s_\theta^4 \Pi_{Z+}^W + \Pi_{h+}^W + \Pi_{2+}^W + \Pi_f^W \quad , \\
(4\pi)^2 \Pi_{WA}^W &= - \left\{ \frac{13}{3} \left(\frac{2}{\varepsilon} - \ln \frac{m_W^2}{\mu^2} \right) + F_1(a_W, 0) \right\} \quad , \\
(4\pi)^2 \Pi_{WZ}^W &= - \left\{ \frac{13}{3} \left(\frac{2}{\varepsilon} - \ln \frac{m_W m_Z}{\mu^2} \right) + F_1(a_W, a_Z) \right\} \quad , \\
(4\pi)^2 \Pi_{Wh}^W &= -a_W F_2(a_W, a_h) \quad , \\
(4\pi)^2 \Pi_{A+}^W &= -a_W F_2(0, 0) \quad , \\
(4\pi)^2 \Pi_{Z+}^W &= -a_Z F_2(a_Z, 0) \quad , \\
(4\pi)^2 \Pi_{h+}^W &= \frac{1}{12} \left(\frac{2}{\varepsilon} - \ln \frac{m_h^2}{\mu^2} \right) + F_3(a_h, 0) \quad , \\
(4\pi)^2 \Pi_{2+}^W &= \frac{1}{12} \left(\frac{2}{\varepsilon} - \ln \frac{-p^2}{\mu^2} + \frac{8}{3} \right) \quad , \\
(4\pi)^2 \Pi_f^W &= \frac{1}{3} \sum_f \left(\frac{2}{\varepsilon} - \ln \frac{m_{f1} m_{f2}}{\mu^2} + F_f(a_{f1}, a_{f2}) \right) \quad ,
\end{aligned} \tag{15}$$

$$\begin{aligned}
(4\pi)^2 \Gamma^{(3)} &= \frac{9}{4} \left(\frac{2}{\varepsilon} - \ln \frac{-p^2}{\mu^2} + \frac{4}{3} \right) \quad , \\
\Pi^{(3)} &= \Pi_{gg}^{(3)} + \Pi_f^{(3)} \quad , \\
(4\pi)^2 \Pi_{gg}^{(3)} &= -\frac{13}{2} \left(\frac{2}{\varepsilon} - \ln \frac{-p^2}{\mu^2} + \frac{97}{78} \right) \quad , \\
(4\pi)^2 \Pi_f^{(3)} &= \frac{2}{3} \sum_f \left(\frac{2}{\varepsilon} - \ln \frac{m_f^2}{\mu^2} + F_f(a_f, a_f) \right) \quad ,
\end{aligned} \tag{16}$$

where we have introduced the following abbreviations

$$\begin{aligned}
a_i &= \frac{m_i^2}{-p^2} \quad , \quad s_\theta^2 = \sin^2 \theta_W \quad , \quad c_\theta^2 = 1 - s_\theta^2 \quad , \\
\frac{2}{\varepsilon} &= \frac{2}{n-4} - \gamma + \ln 4\pi \quad .
\end{aligned}$$

The self-energies above come from the diagrams of Figs. (1) and (2). The sliding mass scale is denoted by μ , and n is the dimension of the space-time. Notice that the effective charges do *not* depend on μ , but only on the masses and the external momenta. The F_i functions contain the threshold effects for particle production. The functions $F_i(a_j)$, $i = 1, 3, f$, behave like $A_i \ln a_j$ when a_j goes to zero, and they tend to a constant value when a_j goes to infinity. On the other hand, $F_2(a_j) \rightarrow 0$ in both limits. Therefore, in the high energy limit, $a_i \rightarrow 0$, we recover the \overline{MS} -expressions:

$$\frac{1}{g_i^2(\mu^2)} = \frac{1}{g_i^2(m_Z^2)} + \beta_i \ln \frac{m_Z^2}{\mu^2} , \quad (17)$$

where, for 3 generations of fermions and 1 scalar doublet

$$(4\pi)^2 \beta_e = \frac{11}{3} , \quad (4\pi)^2 \beta_2 = -\frac{19}{6} , \quad (4\pi)^2 \beta_3 = -7 . \quad (18)$$

Equation (12) allows us to study the evolution of the couplings from m_Z to higher energies, and therefore to extract consequences concerning perturbative unification, taking into account all the caveats and ambiguities discussed above. The input values we will use, at the scale m_Z , are [11]:

$$\begin{aligned} \sin^2 \theta_W &= 0.2329 \pm 0.0013 , \\ \alpha_e^{-1} &= 127.9 \pm 0.3 , \\ \alpha_3 &= 0.111 \pm 0.003 , \\ m_W &= 80.6 \pm 0.4 \text{ GeV} , \quad m_Z = 91.161 \pm 0.031 \text{ GeV} . \end{aligned}$$

Due to the embedding of the SM in $SU(5)$, one normalizes the $U(1)_Y$ coupling as [2]:

$$\alpha_1^{-1} = \frac{3}{5} \alpha'^{-1} = \frac{3}{5} (\alpha_e^{-1} - \alpha_2^{-1}) .$$

Since the top and the Higgs have not yet been detected, we will take their masses as free parameters, whose lower experimental bounds will be set as : $m_t \geq 90 \text{ GeV}$, $m_h \geq 48 \text{ GeV}$ [12]. On the other hand, these masses can not be much higher than about 200 GeV as required by perturbative bounds [13]. This is the value we will adopt for both m_t and m_h ;

however, it can be seen that values between $100 - 1000 \text{ GeV}$ give rise to the same *qualitative* behavior.

In Fig. (3) we have plotted $\alpha_i^{-1}(\mu)$ calculated with the three 1-loop procedures previously discussed: mass dependent (MD) procedure (eq. (12)), integration with θ functions, and \overline{MS} . Only the high energy region ($10^{12} - 10^{19} \text{ GeV}$), where unification⁴ could occur, is depicted. We see at once that the approximation we use for α_1^{-1} and α_3^{-1} make some difference although in this case very little: for example, α_3^{-1} -MD is a little larger than the other couplings, due to the fact that when thresholds are taken into consideration the top quark decouples at low energies. This effect propagates to high energies by the RGE and in this case the decoupling is smoother than in the θ approximation.

In the running of α_2^{-1} a larger number of massive particles participate, and therefore their decoupling will introduce larger corrections than for the other two couplings. This is readily seen in the figure, which also shows that α_2^{-1} - θ is a little larger than α_2^{-1} - \overline{MS} (same reason as for α_3^{-1}), and they both are larger than α_2^{-1} -MD. We now have the contributions of W^\pm and Z^0 , which in the mass dependent scheme take “longer” to decouple. This is the dominant decoupling effect in α_2^{-1} , even though $m_Z, m_W \leq m_t, m_h$, which is not operative in α_2^{-1} - θ because we begin to integrate precisely at the scale of m_Z . That is, the thresholds effects tend to make α_2^{-1} “less asymptotically free”. This was already pointed out in Refs. [14], [15] which show that, within the context of the minimal $SU(5)$ model, the β -function for the $SU(2)$ coupling with thresholds is positive in the region around m_W .

We have also calculated α_i^{-1} at 2-loop order *without* thresholds. The 2-loop RGE's in the \overline{MS} are well known [16]; solving these equations by an iterative technique [8,6], one obtains:

$$\alpha_i^{-1}(\mu) = (\alpha_i^{-1}(\mu))^{(1)} + \frac{b_{ij}}{(4\pi)b_j} \ln \frac{\alpha_j(m_Z)}{\alpha_j(\mu)} , \quad (19)$$

⁴Of course this is not the case for the SM. This plot is simply intended to illustrate how the decoupling affects the evolution of the coupling constants.

$$\alpha_i^{-1}(\mu)^{(1)} = \alpha_i^{-1}(m_Z) + \frac{b_i}{(4\pi)} \ln \frac{m_Z^2}{\mu^2} ,$$

where $i, j = 1, 2, 3$; the coefficients b_i, b_{ij} for 3 generations of fermions and 1 scalar doublet are:

$$b_i = \begin{pmatrix} 41/10 \\ -19/6 \\ -7 \end{pmatrix} , \quad b_{ij} = \begin{pmatrix} 199/50 & 27/10 & 44/5 \\ 9/10 & 35/6 & 12 \\ 11/10 & 9/2 & -26 \end{pmatrix} . \quad (20)$$

In this case we calculate directly α_1^{-1} , rather than α_e^{-1} , because working in \overline{MS} is equivalent to working with the symmetric theory $SU(3) \times SU(2) \times U(1)$.

In Fig. (4) we can see that 2-loop effects are qualitatively similar to MD effects: they raise α_3^{-1} and decrease α_2^{-1} , even though in the case of α_2^{-1} the effect from the decoupling of the gauge bosons dominates over 2-loop effects. Thus, if we take into account both of them (2-loop and thresholds) by calculating α_i^{-1} with an MDSP at 2-loop order, α_2^{-1} will be less asymptotically free than $\alpha_2^{-1}-\overline{MS}$ at 1-loop.

Short of doing the exact calculation we have studied this case with an appropriate approximation for the threshold functions. We approximate $\alpha_i^{-1(1)}$, b_i, b_{ij} in Eq. (19) by:

$$\alpha_i^{-1}(\mu)^{(1)} = \alpha_i^{-1}(m_Z) + \frac{1}{(4\pi)} \sum_k b_i^{(k)} \left(f^{(k)}(a_k) \big|_{(-p^2=m_Z^2)} - f^{(k)}(a_k) \big|_{(-p^2=\mu^2)} \right) , \quad (21)$$

$$b_i = \sum_k b_i^{(k)} f'^{(k)}(a_k) , \quad (22)$$

$$b_{ij} = \sum_l \sum_k b_{ij}^{(k,l)} f'^{(k)}(a_k) f'^{(l)}(a_l) , \quad (23)$$

where we have summed over all the particles in the SM, and [4]:

$$f^{(k)}(a_k) = \ln(1 + c_k a_k) - \ln c_k a_k , \quad (24)$$

$$f'^{(k)}(a_k) = \frac{d f^{(k)}}{d \ln(-p^2)} = \frac{1}{1 + c_k a_k} . \quad (25)$$

These functions $f^{(k)}$ and $f'^{(k)}$ have been chosen so that their behavior in the limit $a_k \rightarrow 0$ is:

$$f^{(k)}(a_k) \rightarrow -\ln c_k a_k \quad , \quad f'^{(k)}(a_k) \rightarrow 1 \quad , \quad (26)$$

and both functions vanish when $a_k \rightarrow \infty$. Thus in the high energy limit (or for massless particles) we recover the $\alpha_i^{-1}-\overline{MS}$ expressions; the heavy masses decouple at low energy more smoothly than in the decoupling model by a step function. This approximation is very good at 1-loop order: comparing the numerical results with those obtained from α_i^{-1} -MD, the differences are only of order 0.1%. We also expect this to be true at 2-loop order. Actually it is not necessary to modify the 2-loop coefficient b_{ij} , since there are no appreciable differences if we take b_{ij} as given by the \overline{MS} prescription or as given by Eq.(23). This may suggest that 2-loop thresholds are not relevant (only 1-loop thresholds are). In fact, 2-loop threshold effects are quite small since they are corrections to the 2-loop \overline{MS} contribution which is, in turn, a correction to the 1-loop contribution.

If we compare the 1-loop and 2-loop results, both within this approximation, we see (Fig. 5) that “2-loop+thresholds” has the effect of lowering α_2^{-1} and raising α_3^{-1} . Therefore, combined effects of mass and order of the perturbation theory, tend to bring α_2^{-1} and α_3^{-1} *closer* at high energies. Unfortunately this effect is not strong enough to unify the three couplings of the SM, but it is interesting to point out its existence.

It is clear this effect will become the more relevant the larger the number of massive particles we have. So far we have only worked with the standard matter content of the SM, in which all the particles have been detected, and their masses measured, except for the top and the Higgs. The particle contents can be modified, for example by extending the SM and working in the so called Minimal Supersymmetric Standard Model (MSSM). Here more degrees of freedom (the susy partners etc.) begin to contribute between m_Z and the putative unification scale. Since these susy particles have not been detected, there are at most lower bounds available to their masses. We can make use of some relations between them which reduce the number of the necessary arbitrary mass parameters [17]. The common mass for both squarks and sleptons, m_0 , and the gaugino mass, $m_{1/2}$, may be bounded by demanding that susy masses be in the range between 45 *GeV* and 1 *TeV*.

We also take $m_Z \leq m_\mu$, $m_+ \simeq m_H \leq 1 \text{ TeV}$, and $m_t, m_h \simeq 200 \text{ GeV}$, where m_μ is the higgsino mass.

Previous works in MSSM indicate that an $M_X \approx O(10^{16})$ and a common M_{Susy} (or explicit susy masses) in the range from m_Z to $O(1\text{TeV})$ are compatible with experimental bounds on proton decay [11,18,19]. In these analyses, and except for the case of 1-loop \overline{MS} , the values for $\alpha_3(m_Z)$ larger than 0.111 are favoured to have an M_{Susy} in this range. The latest data on $\alpha_3(m_Z)$ from LEP (see table 1) [20] indicate an $\alpha_3(m_Z)$ larger than this by 12%. Thus, until we have a more precise determination of $\alpha_3(m_Z)$ we will have to take the allowed range for $\alpha_3(m_Z)$ to be (0.108, 0.125). Systematically one sees that the higher the value of $\alpha_3(m_Z)$ the higher M_X , but the lower M_{Susy} . This systematic also happens when we use 2-loop instead of 1-loop RGE's [19]. The trend is maintained using Mass Dependent RGE's, but *now* the values of the susy masses needed to unify the couplings are *higher* than with the other methods.

In Fig. (6) we have represented α_i^{-1} -MD for two different values of the susy parameters and $\alpha_3(m_Z) = 0.111$. We have also included α_i^{-1} - \overline{MS} with $M_{Susy} = m_Z$ and the same value of $\alpha_3(m_Z)$ in order to compare the results yielded by different approximations. As pointed out before, the α_i^{-1} - \overline{MS} unify, while the α_i^{-1} -MD do not. For example, we would need susy masses of order 10^2 TeV for $\alpha_3(m_Z) = 0.111$, and of order 1 TeV for $\alpha_3(m_Z) = 0.120$. With $\alpha_3(m_Z)$ in the range given before, and requiring $M_X \simeq O(10^{16})$ to avoid conflicts with proton decay, susy masses of order m_Z are excluded, except $m_{1/2} \simeq m_{\tilde{w}}$ which has to be less than $O(3\text{TeV})$. At 2-loop order, with complete light thresholds at 1-loop, the bounds increase: $m_{1/2}$ greater than approximately 3.5TeV is excluded, but the rest of susy masses must be greater than $O(10\text{TeV})$.

So far, we have discussed perturbative unification within MSSM, but without any reference to the unification group G , and the new heavy fields which are introduced by G in the theory. Moreover, we have required that the couplings “unify at the scale M_X ” when they really “cut at the scale M_X ” and run separately after it. To speak properly about perturbative unification, we need that the couplings converge to only a single running coupling

constant: the G coupling. We would obtain this when we work within G , the group $SU(5)$, taking into account the thresholds of all the masses, both light and heavy masses [14,21]. As we have seen, light thresholds introduce appreciable differences in the running of the couplings, and the same effect takes place with the heavy masses when the scale approaches M_X [22]. Although we will have many more free parameters (the heavy masses), the trend is the same that in the simplest case when no reference was made to G : with mass dependent RGE's we need heavier susy masses in order to obtain unification with $\alpha_3(m_Z)$ in the range allowed by experimental data.

In conclusion, we have studied the evolution of the running couplings of the SM including complete threshold effects due to the light particles. We have also used effective charges [10], which are *mass dependent*, and *process* and *gauge* independent when we choose an adequate initial condition for the gauge parameter (Landau gauge). We have also seen that the effect of the decoupling of the light masses is *not* negligible at high energy. We apply the same method to study the MSSM, and the unification of the couplings within this model. Finally, We have found that light thresholds have the property of raising the values of the susy masses needed to keep $\alpha_3(m_Z)$ within the range of the already available experimental values.

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REFERENCES

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- [1] H. Georgi and Glashow, Phys. Rev. Lett. 32 (1974) 438.
- [2] H. Georgi, H. R. Quinn and S. Weinberg, Phys. Rev. Lett. 33 (1974) 451.
- [3] W. A. Bardeen, A. Buras, D. Duke and T. Muta, Phys. Rev. D 18 (1978) 3998.
- [4] H. Georgi and H. D. Politzer, Phys. Rev. D 14 (1974) 451.
- [5] T. Appelquist and J. Carrazone, Phys. Rev. D 11 (1975) 2856.
- [6] W. J. Marciano and A. Sirlin, *in* Proc. of the Second Workshop on Grand Unification, Ann Arbor, 1981, Eds. J. Leveille, L. Sulak and D. G. (Birkhauser, Boston).
- [7] N-P Chang, A. Das and J. P. Mercader, Phys. Lett. B 93 (1980) 137; S. Weinberg, Phys. Lett. B 91 (1980) 51; C. H. Llewellyn Smith, G. G. Ross and J. F. Wheeler, Nucl. Phys. B 177 (1981) 263.
- [8] L. Hall, Nucl. Phys. B 178 (1981) 75.
- [9] M. Gell–Mann and F. E. Low, Phys. Rev. 95 (1954) 1300; E. C. G. Stueckelberg and A. Petermann, Helv. Phys. Acta 26 (1953) 499.
- [10] D. C. Kennedy and B. W. Lynn, Nucl. Phys. B 322 (1989) 1.
- [11] J. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. B 260 (1991) 131.
- [12] Particle Data Book
- [13] N. Cabbibo, L. Maiani, G. Parisi and R. Petronzio, Nucl. Phys. B 158 (1979) 295; R.

- A. Flores and M. Sher, Phys. Rev. D 27 (1983) 1679; M. Lindner, Z. Phys. C 31 (1986) 295.
- [14] D. A. Ross, Nucl. Phys. B 140 (1978) 1.
- [15] T. Goldman and D. A. Ross, Nucl. Phys. B 171 (1980) 301.
- [16] M. B. Einhorn and D. R. T. Jones, Nucl. Phys. B 196 (1982) 475.
- [17] A. B. Lahanas and D. V. Nanopoulos, Phys. Reports 145 (1987) 1.
- [18] J. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. B 249 (1990) 441; U. Amaldi, W. de Boer and H. Furstenau, Phys. Lett. B 260 (1991) 447; P. Langacker and M. Luo, Phys. Rev. D 44 (1991) 817; F. Anselmo, L. Cifarelli, A. Peterman and A. Zichichi, Nuovo Cimento A 104 (1991) 1917; V. Barger, M.S. Berger and P. Ohmann, preprint MAD-PH-711 (1992); J. Ellis, S. Kelley and D. V. Nanopoulos, Nucl. Phys. B 373 (1992) 55; F. Anselmo, L. Cifarelli, A. Peterman and A. Zichichi, Nuovo Cimento A 105 (1992) 1201.
- [19] F. Anselmo, L. Cifarelli, A. Peterman and A. Zichichi, Nuovo Cimento A 105 (1992) 581; Nuovo Cimento A 105 (1992) 1179; J. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. B 287 (1992) 95; G. G. Ross and R. G. Roberts, Nucl. Phys. B 377 (1992) 571; M. Carena, s. Pokorski and C.E.M. Wagner, preprint MPI-Ph/93-10 (1993); P. Langacker and N. Polonsky, Phys. Rev. D 47 (1993) 4028.
- [20] ALEPH Collab., D. Decamp et. al., Phys. Lett. B 284 (1992) 163; DELPHI Collab., P. Abreu et. al., Z. Phys. C 54 (1992) 55; L3 Collab., O. Adriani et. al., Phys. Lett. B 284 (1992) 471; OPAL Collab., P. D. Acton, Z. Phys. C 55 (1992) 1; A. X. El Khadra, G. Hockney, A. S. Kronfeld and P. B. Mackenzie, Fermilab preprint PUB-91/354-T (1991); W. Wong, P. B. Mackenzie, R. Rosenfeld and J. L. Rosner, Phys. Rev. D 37 (1988) 3210; A. D. Martin, R. G. Roberts and W. J. Stirling, Durham preprint DTP 90-76 (1990); Rutherford preprint RAL-91-044 (1991).

- [21] F. Anselmo, L. Cifarelli, A. Peterman and A. Zichichi, *Nuovo Cimento* **A 105** (1992) 1025; *Nuovo Cimento A* 105 (1992) 1357; J. Hisano, H. Murayama and T. Yanagida, *Phys. Rev. Lett.* 69 (1992) 1014; *Nucl. Phys. B* 402 (1993) 46.
- [22] This is now investigated in a paper in preparation.

TABLES

Experiment	Central value	Error
ALEPH jets	0.125	± 0.005
DELPHI jets	0.113	± 0.007
L3 jets	0.125	± 0.009
OPAL jets	0.122	± 0.006
OPAL τ	0.123	± 0.007
J/Ψ	0.108	± 0.005
Υ	0.109	± 0.005
Deep Inelastic	0.109	± 0.005

TABLE I. Experimental values of $\alpha_3(m_Z)$

FIGURES

FIG. 1. The 1-loop particle contributions to the $\Pi^{(j)}$ functions ($j = A; W^\pm$). (a) Pure gauge boson contributions plus ghost (ω_i) contributions. (b) Graphs that mix scalars and gauge bosons. (c) Pure scalar contributions. (d) Fermion contributions. ϕ_1 stands for the physical Higgs scalar; ϕ^\pm , ϕ_2 for the Goldstone bosons associated with W^\pm and Z ; f for fermions (left and right); and f_{1L} , f_{2L} for the components of each doublet of left fermions.

FIG. 2. The 1-loop particle contributions to the Π_{ZA}^L function, *i.e.* to Γ' . (a) Gauge boson plus ghost. (b) Scalars and gauge bosons.

FIG. 3. Evolution of the three couplings of the SM calculated with the three 1-loop procedures: mass dependent (solid lines), θ function (dashed lines), and \overline{MS} (dotted lines).

FIG. 4. Evolution of the three couplings of the SM at 2-loop order with \overline{MS} (solid lines), and 1-loop order with a mass dependent method (dashed lines). We also plot 1-loop \overline{MS} for comparison (dotted lines).

FIG. 5. Evolution of the three couplings of the SM at 1-loop order (solid lines), and 2-loop order (dashed lines), both with approximated threshold functions.

FIG. 6. Evolution of the three couplings of the MSSM at 1-loop order calculated with (a) \overline{MS} with $M_{susy} = m_Z$ (solid lines); (b) MD with $m_{1/2} = 45 \text{ GeV}$ and $m_0 = m_Z$ (dashed lines); (c) MD with $m_{1/2} = m_0 = 1 \text{ TeV}$ (dotted lines); we take $m_\mu = m_\tau = m_H = m_0$ and $m_t = m_b = 200 \text{ GeV}$.

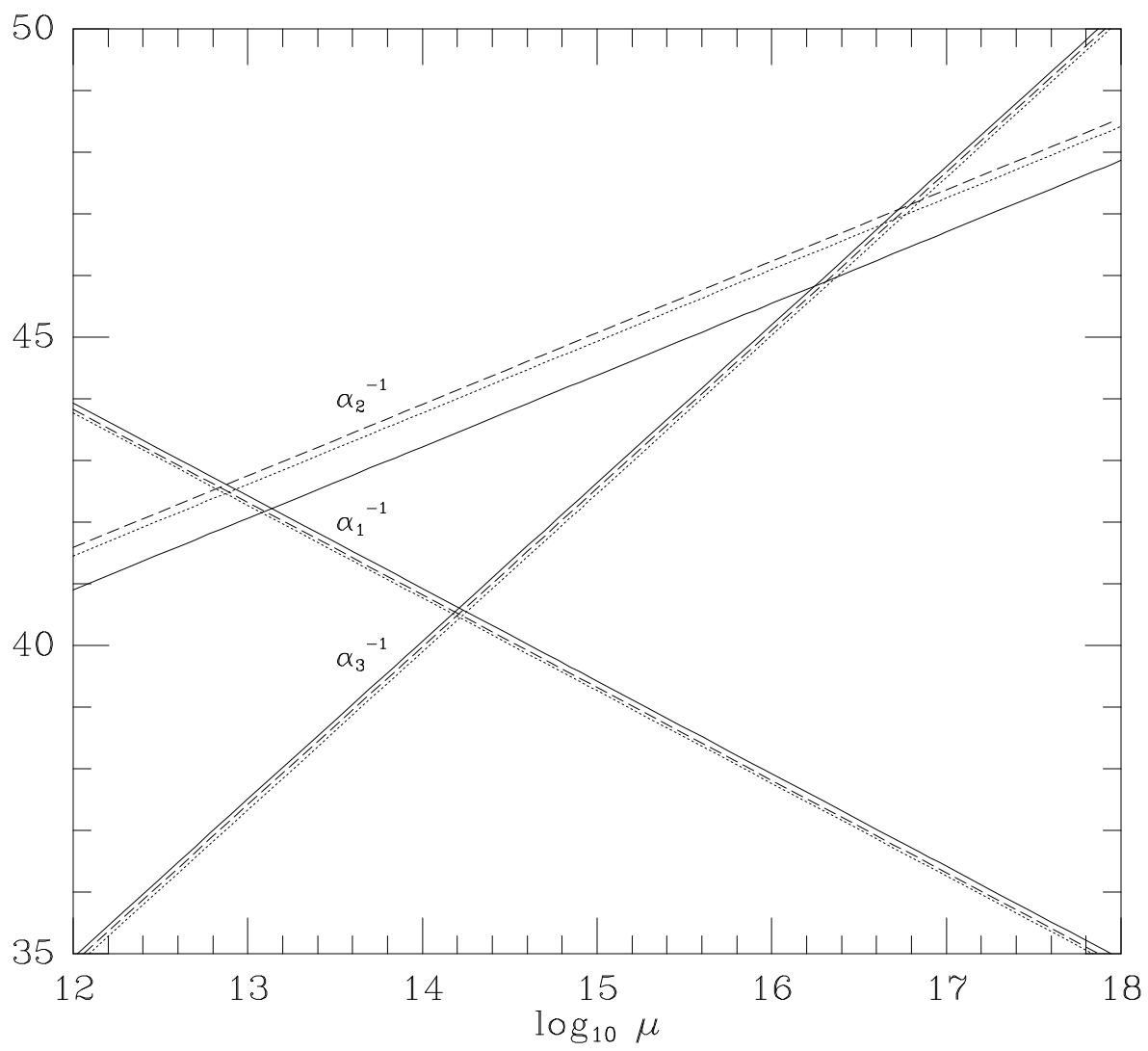


Figure 3

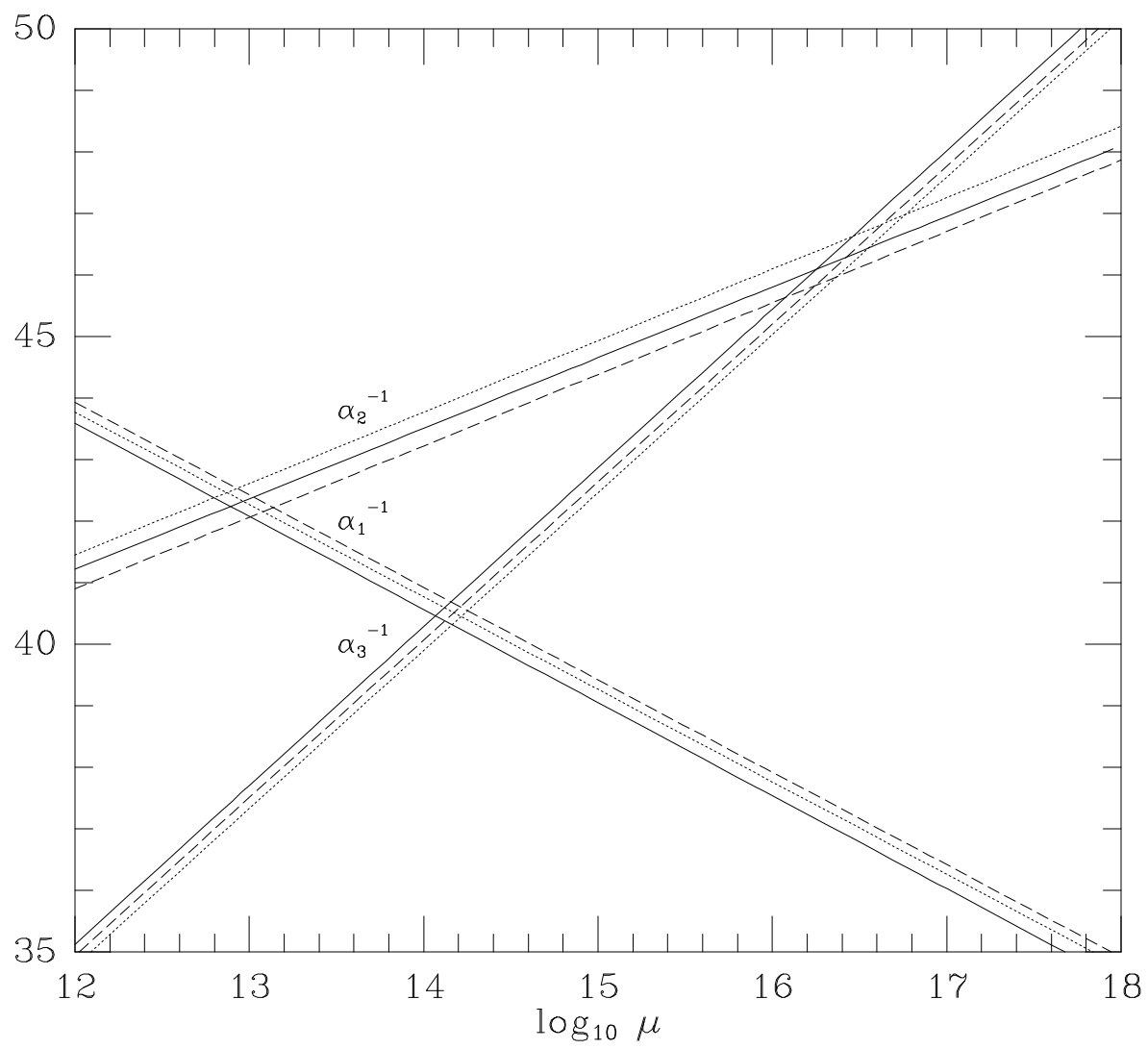


Figure 4

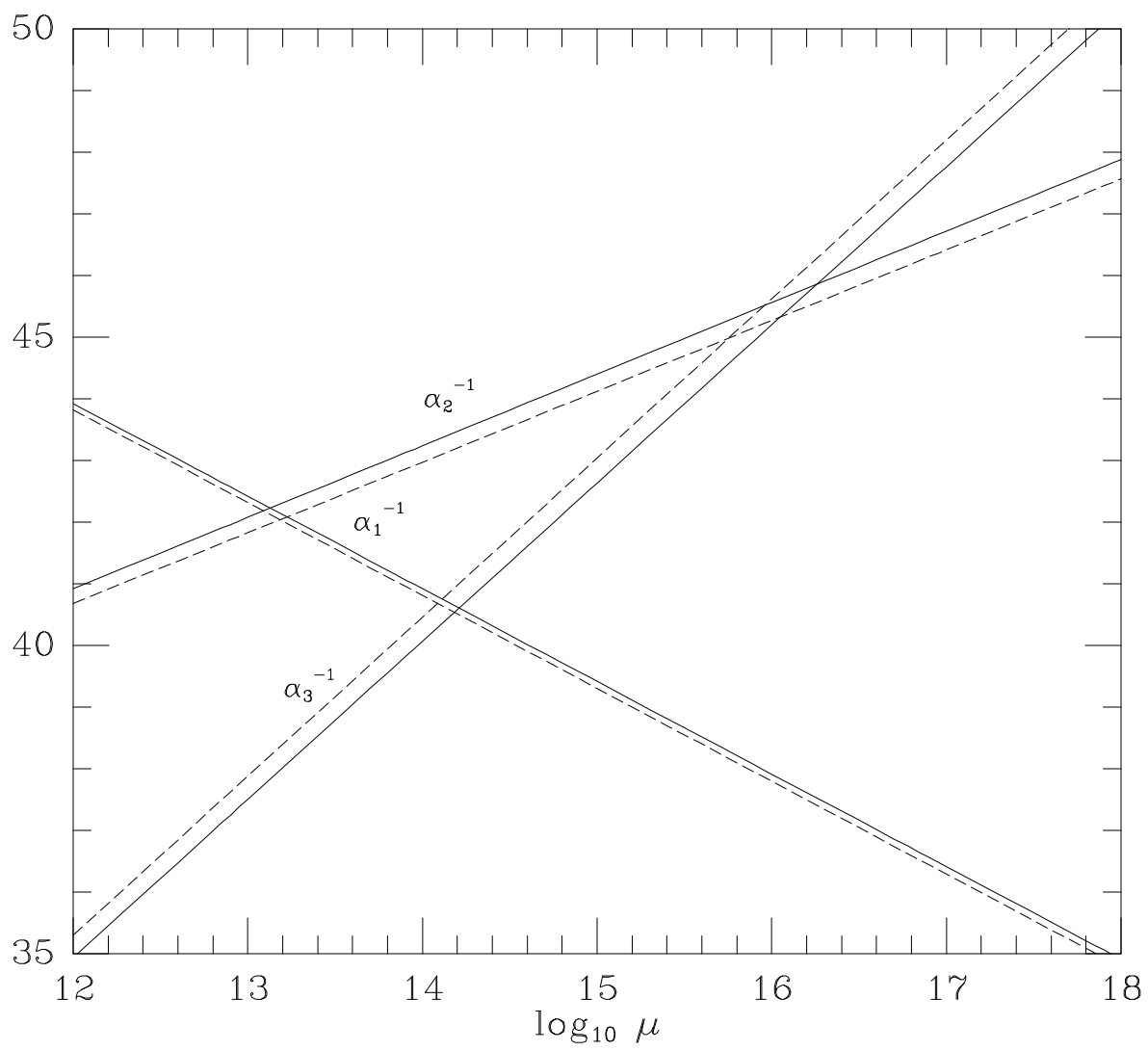


Figure 5

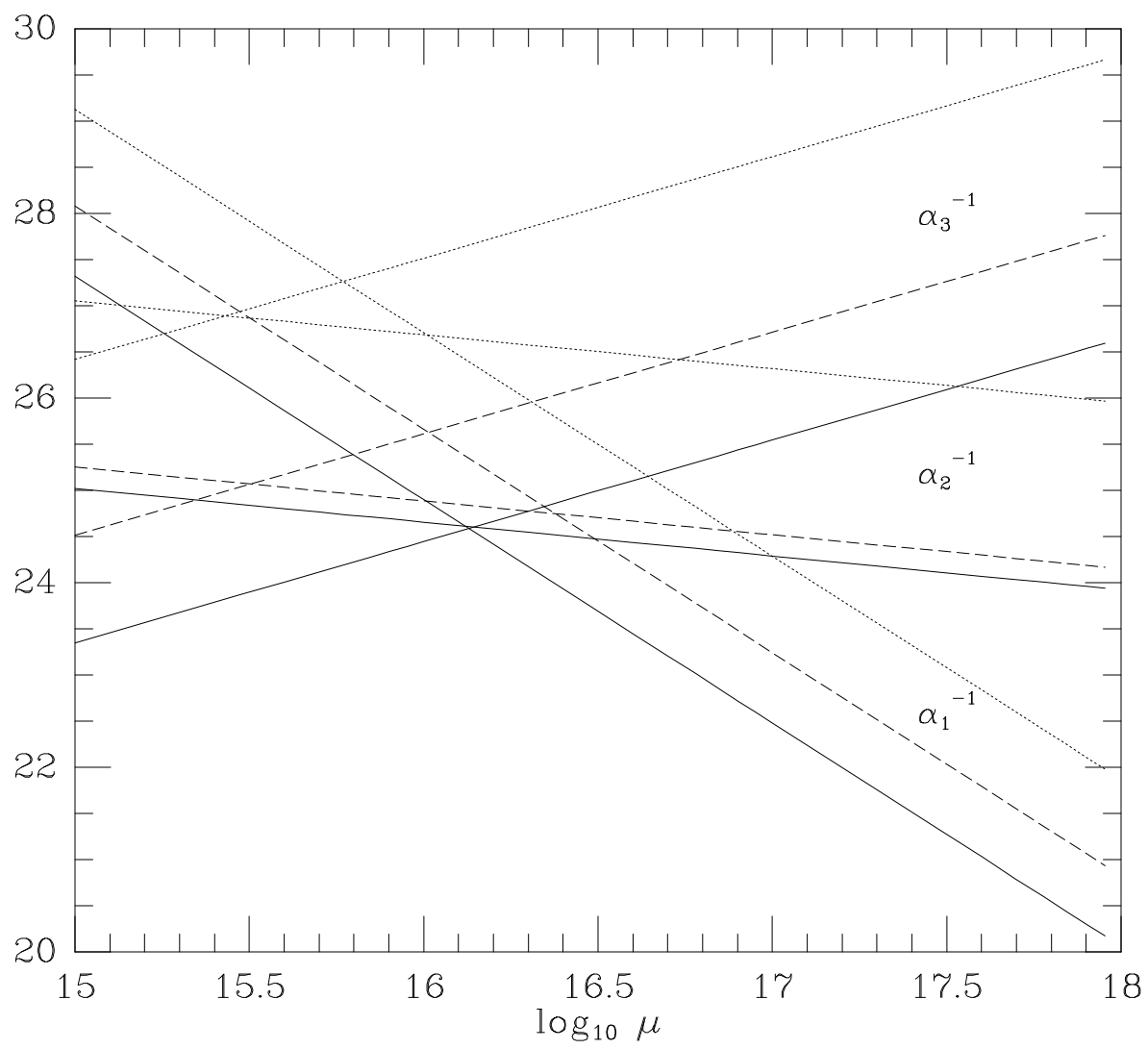


Figure 6

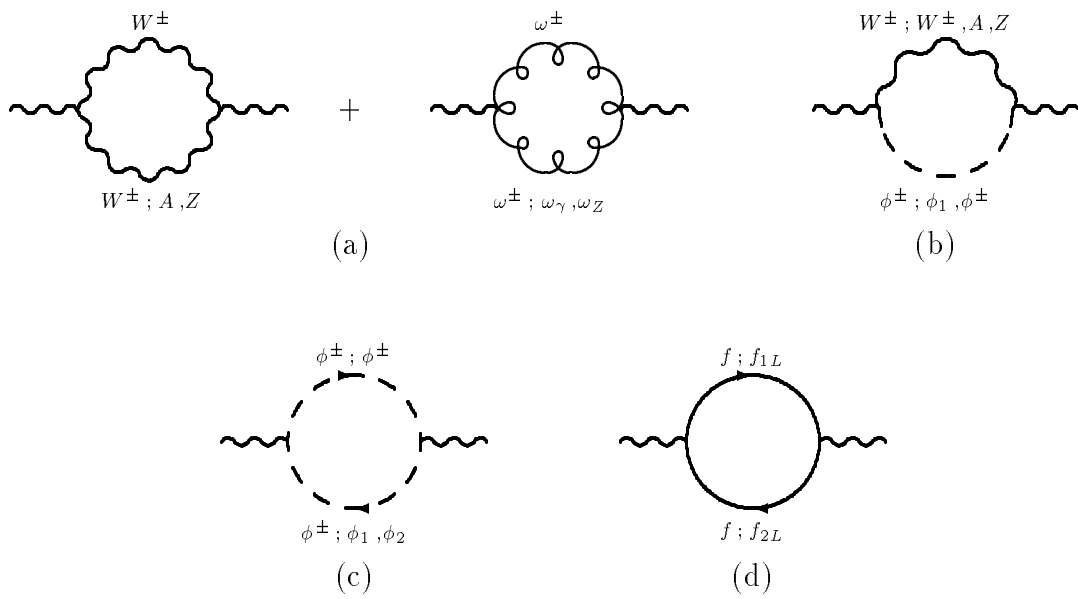


Figure 1

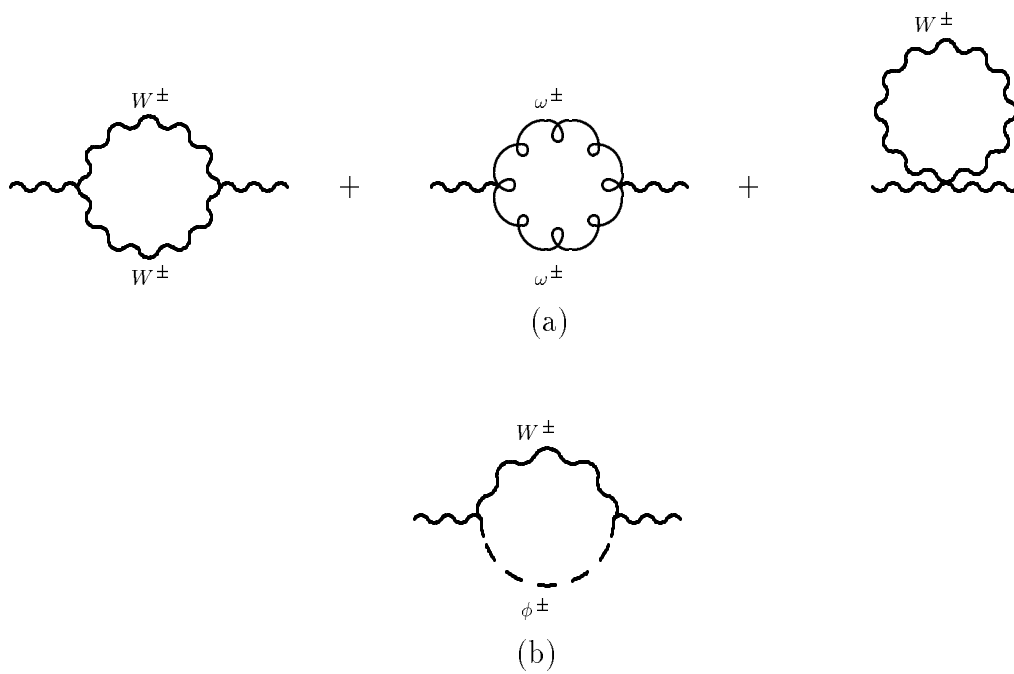


Figure 2